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Uncertainty assessment in friction factor measurements as a tool to design experimental set-ups

Marco Lorenzini [a,*], Gian Luca Morini [a], Torsten Henning [b], Jürgen Brandner [c]

[a] DIENCA Università di Bologna, Viale Risorgimento 2, I-40136 Bologna, Italy
[b] Institute for Experimental Physics I, Justus Liebig University, Heinrich-Buff-Ring 16, DE-35392 Giessen, Germany
[c] Forschungszentrum Karlsruhe, Institute for Micro Process Engineering IMVT, Hermann-von-Helmholtz-Platz 1, DE-76344 Eggenstein-Leopoldshafen, Germany

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Abstract

The promising performance of microchannels has given rise to intensive research on pressure drop and heat transfer characteristics of flows at the small scale. To check the classical models and to validate new ones, experiments need to be conducted, which are particularly difficult given the characteristic dimensions involved and the magnitude of the fluxes to be measured. Although more care has been devoted lately to the design of experiments in terms of control of the geometry and of the boundary conditions, the uncertainties which inevitably affect each measurement do not seem to have been given the proper consideration. Correctly calculating uncertainties not only allows a correct assessment of the experimental data obtained, but can also be used to decide which measurements need to have the highest precision to achieve a certain accuracy, thus saving money on the others. In this paper, a quantitative criterion is given to assess the accuracy achievable in the determination of the friction factor in the laminar regime for the flow of a fluid in a circular microtube. The influence of the six quantities (pressure drop, outlet pressure, temperature, length, diameter and volume flow rate) measured to determine f in the laminar regime are studied separately and when combined. It is found that at low Reynolds numbers flow rate and pressure drop measurements are determinant for the final value of the uncertainty, while at larger Reynolds numbers the influence of the accuracy in measuring the hydraulic diameter prevails and also limits the minimum value that the total uncertainty can take.

Keywords: Uncertainty analysis; Friction factor; Microchannel gas flow

1. Introduction

Ever since the pioneering works of Tuckermann and Pease [1,2] in the early 80s, microchannels have enjoyed an increasing and enduring interest from the scientific and technological community, largely owing to the increased performance in terms of mass, energy and chemical species transport in comparison to their macroscopic counterparts.

The earliest experimental works [3–5] aimed at characterising the behaviour of either single or multiple channels seemed to evidence completely different values of such parameters as friction factor and heat transfer coefficient, while the newly introduced correlations failed most of the time to predict the data except those which were used to obtain them, and all sorts of effects were called upon to explain the apparently anomalous behaviour (e.g., the electric double layer, roughness, etc.).

As further studies were carried out, the awareness of the necessity of carefully controlled experimental conditions grew and the dramatic progress in fabrication techniques which increased the geometric accuracy of the tested specimens reduced considerably the difference between micro- and macrochannel behaviour. In many cases it was proven that the discrepancies are to be ascribed to scaling effects [6,7] which are negligible in the macroscale but must be accounted for when the characteristic dimensions decrease.

If the effects which are important at the microscale are to be singled out, the experiments must possess an adequate accuracy, or the uncertainties might be larger than the deviations from

* Corresponding author. Tel.: +39 051 2093293; fax: +39 051 2093286.
E-mail address: marco.lorenzini@unibo.it (M. Lorenzini).

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Nomenclature

A_c	cross-sectional area	m^2
C_{in}	loss coefficient at the inlet	
C_{ex}	loss coefficient at the outlet	
D	hydraulic diameter	m
f	friction factor	
fRe	Poiseuille number	
F	generic quantity	
L	length of the microchannel	m
m	mass flow rate	$kg\,s^{-1}$
N_2	exponent for the fluid	
p_1	pressure at the inlet tap	Pa
p_2	pressure at the outlet tap	Pa
p_{in}	pressure at the inlet	Pa
p_{ex}	pressure at the outlet	Pa
R	gas constant	$J\,kg^{-1}\,K^{-1}$
Re	Reynolds number	
T	fluid temperature	K
T_m	measured fluid temperature	K
T_r	reference fluid temperature	K
U_F	absolute uncertainty on F	varies
$\dot{V}$	volume flow rate	$m^3\,s^{-1}$
Δp_d	pressure drop due to flow development	Pa
Δp_{ex}	pressure drop at the outlet	Pa
Δp_{in}	pressure drop at the inlet	Pa
Δp_m	total pressure drop	Pa
Δp_n	net pressure drop	Pa
δF	relative uncertainty on F	
μ	dynamic viscosity	Pa s
μ_a	reference dynamic viscosity	Pa s

the traditional behaviour which one aims at evidencing, as was demonstrated in a previous paper [8].

It is the authors' opinion that experimental uncertainty deserves being more directly addressed, in order to evidence its potential not only as a tool to assess the accuracy and thus the relevance of obtained experimental data, but also as a predictive tool to help in deciding the precision (which corresponds in practice to a cost) of the instrumentation to set up an experiment, thus avoiding unnecessary expenses to obtain highly accurate measurements of quantities whose uncertainty is only of marginal influence in determining the parameters sought.

The practical application which is considered here is the calculation of the friction factor for the isothermal, laminar flow of gases within circular microchannels, but it can be extended to other geometries and fluids and to the case of heat transfer measurements.

The uncertainty analysis is a known tool which is currently employed quite widely in many engineering applications, but there is still some unclearness in its use. If some works like those of Li [9], Celata et al. [10], Park and Punch [11] do report the final uncertainty on the calculated quantities, there are some other, like those of Yang and El-Genk [12] where the given uncertainties on the measured quantities are not coherent with the final uncertainty value of the calculated friction factor, without any explanation being given; moreover it is uncommon to find both the uncertainties of the measured and of the calculated quantities, which should always be given.

2. The friction factor coefficient

When a gas flows isothermally in a microchannel of circular cross section, the friction factor can be calculated from Eq. (1) [13] as:

$$f = \frac{D}{L}\left[\left(\frac{1-(1-\Delta p_n/p_{in})^2}{(\dot{m}\sqrt{RT}/A_c p_{in})}\right) - 2\ln\left(\frac{1}{1-\Delta p_n/p_{in}}\right)\right] \quad (1)$$

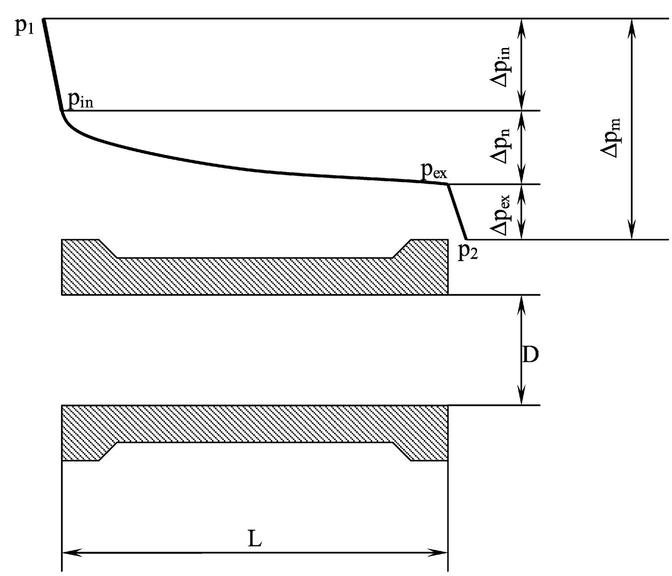


Fig. 1. Pressure drop distribution within a microchannel.

where the meaning of the symbols is explained in the nomenclature. The above equation can be used for both compressible and incompressible flows: it can be shown that in the latter case the expression for f is the same except for the logarithmic term, which goes to zero in (1) as compressibility effects disappear. Not all the quantities appearing in Eq. (1) can normally be directly measured: in particular, the inlet pressure p_{in} and the net pressure difference result often from other measurements. In many cases, in fact, the pressure difference is measured at two points before the inlet of the microchannel and past its outlet, and the absolute value of the pressure at one of the two measurement points must also be known.

Looking at Fig. 1, the net pressure drop, Δp_n is calculated in general from the measured pressure drop $\Delta p_m = p_1 - p_2$, as

$$\Delta p_n = \Delta p_m - \Delta p_{in} - \Delta p_{ex} \quad (2)$$

The pressure losses at the inlet and at the outlet amount to:

$$\Delta p_{\text{in}} = p_1 - p_{\text{in}} = \frac{C_{\text{in}}}{2}\left(\frac{\dot{m}}{A}\right)^2\left(\frac{RT}{p_{\text{in}}}\right) \tag{3}$$

$$\Delta p_{\text{ex}} = p_{\text{ex}} - p_2 = \frac{C_{\text{ex}}}{2}\left(\frac{\dot{m}}{A}\right)^2\left(\frac{RT}{p_{\text{ex}}}\right) \tag{4}$$

with the values of the loss coefficients C_{in} and C_{ex} equal to 1.50 and 0.88, respectively [14].

The friction factor can normally be obtained from the experimental measurement of six quantities, namely:

- microchannel diameter, D;
- microchannel length, L;
- total pressure drop, Δp_m;
- exit pressure, p_2;
- volumetric flow rate, $\dot{V}$;
- fluid temperature, T.

From the knowledge of these quantities it is possible to calculate the friction factor using Eq. (1) and the Reynolds number, which in this case is expressed as:

$$Re = \frac{\dot{m}D}{A\mu} \tag{5}$$

The results are normally plotted on a Moody chart, or the Poiseuille number ($Po = fRe$) is calculated and plotted as a function of the Reynolds number. To assess the accuracy of the results, an uncertainty analysis is usually carried out using the theory of the propagation of error [15]; the uncertainty U_F on a quantity F which is function of $x_1, \ldots, x_n$, measured variables with an associated uncertainty U_{x_i} can be expressed as follows:

$$U_F = \pm\sqrt{\sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}U_{x_i}\right)^2} \tag{6}$$

And the relative uncertainty δF is:

$$\delta F = \frac{U_F}{F} \tag{7}$$

In the case of the friction factor, the relative uncertainty becomes:

$$\delta f = \pm\sqrt{(\delta\alpha)^2 + A(\delta\beta)^2 + B(\delta\gamma)^2 + C(\delta p_{\text{in}})^2} \tag{8}$$

where $\alpha = D/L$, $\beta = \dot{m}/A_c \cdot \sqrt{RT}$, $\gamma = p_{\text{ex}}/p_{\text{in}}$, while A, B and C suitable coefficients which are related to the uncertainties of the measured variables in a rather involved way; the details on the lengthy calculations to obtain them are reported in Appendix A.

Table 1 lists the uncertainties associated with a series of experiments to determine the friction factor for stainless steel and Peeksil microtubes, which have been previously reported in [16,17], and Fig. 2 shows a typical Moody chart with the associated uncertainties: it is to be remarked how the error bars indicating the magnitude of the uncertainty decrease in amplitude as the Reynolds number increases.

Table 1
Relative uncertainties of the measured quantities

Quantity	Range	Uncertainty
Volume flow rate	0–5000 nml/min	±0.5% FS
Volume flow rate	0–500 nml/min	±0.5% FS
Total pressure drop	0–35 kPa	±0.5% FS
	0–86 kPa	
	0–140 kPa	
	0–220 kPa	
	0–860 kPa	
	0–1400 kPa	
Ambient pressure	–	±1%
Tube diameter	0.1–1000 urn	±2%
Tube length	0.25–210 mm	±0.25%
Temperature		±0.2 K

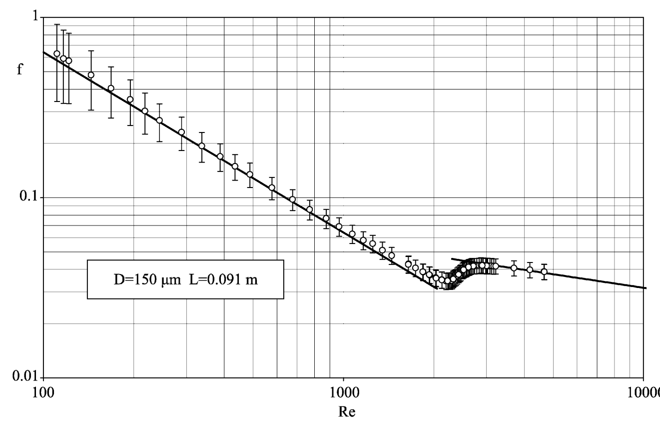


Fig. 2. Moody chart for a Peeksil microtube.

3. Uncertainty analysis as a predictive tool

The uncertainty analysis is indeed an invaluable tool to establish the degree of reliability of a measurement and to validate theoretical models and simulations. Yet, it can also give a tangible contribution to devising experiments. To achieve this goal, one can carry out a sensitivity analysis in order to ascertain which measured quantities need the highest accuracy and which are less affected by the precision of the instrument. From this knowledge, it can be decided if measurements can achieve the precision desired in the first place, and on which ones higher accuracy is to be required.

Such an analysis cannot be applied to extant experimental data, as it must be performed prior to setting up the test rig; moreover, the measured data are inevitably affected by random and bias errors germane to every test, which might change the outcome of the analysis. The problem can easily be solved when a model of the phenomenon to be studied is present, as is the case for the friction factor associated to laminar, isothermal gaseous flow in microtubes: as discussed by Morini et al. [12,13], the Poiseuille law ($fRe = 64$) holds, provided one corrects the data for all contributions. From the knowledge of the friction factor, fixing the gas type (nitrogen) its inlet temperature and the geometrical parameters, as well as the value of the outlet pressure (which in this case coincides with the ambient pressure), the total pressure drop and the volume flow rate

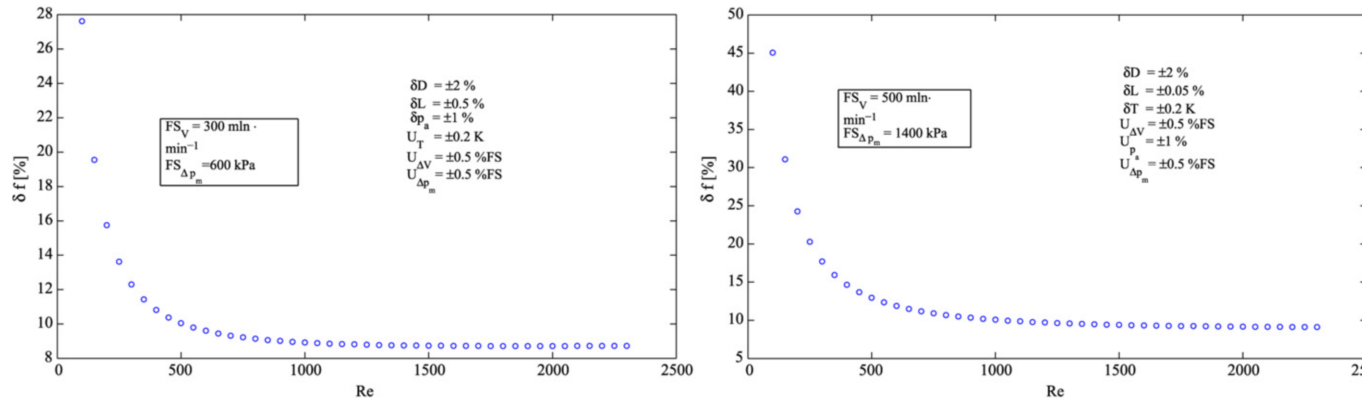


Fig. 3. Influence of the instrumentation's Full Scale ($D = 175$ µm and $L = 50$ cm).

which would be measured in an ideal, error-free experiment are calculated backwards as a function of the imposed Reynolds number using a simple Matlab script.

The temperature, total pressure drop, outlet pressure and volume flow rate are then processed with another Matlab script to set the value of one or more uncertainties on the measured quantities, while those remaining are kept null. Before conducting the real sensitivity analysis, the influence of the full scale of the instruments used for measuring the volume flow rate and pressure drop was investigated, for a microchannel 50 cm long and 175 µm in diameter, through which nitrogen flows. The results for the former case are shown in Fig. 3, where the values of the uncertainties on the measured quantities are also shown (these correspond to values which can be currently achieved without extremely careful experimental set-ups). The increase in accuracy derived from choosing an appropriate scale for the instrumentation is obvious, especially for low values of the Reynolds number: at $Re = 200$, for instance, the uncertainty drops from about 21% to less than 14%. It is also to be noticed that when $Re > 1500$ in the latter case the uncertainty on f exhibits a minimum and then increases very slightly: the reason shall be discussed at the end of this paragraph.

This underlines how the choice of a suitable scale for the instrumentation is crucial: the use of one single transducer for a broad range of variation of the measured quantity out of economical reasons can lead to uncertainties in the data which make the results useless altogether.

The sensitivity to uncertainty of the six measured quantities was then investigated as well as their contribution to the total uncertainty on the friction factor. The set of error-free data was generated for a microtube 175 µm in diameter and 50 cm long, with a 600 kPa full-scale pressure transducer and a flow meter with a maximum capability of 300 mln/min.

Considering values of the uncertainty slightly larger than those normally achieved for the ambient pressure (i.e. $\pm 2\%$), the contribution to the uncertainty on f was modest (a maximum of $\pm 2.4\%$). Similarly, an uncertainty on the measured temperature of ± 0.5 K resulted in a maximum uncertainty of $\pm 2.8\%$ at Reynolds numbers close to transition.

Also the influence of the length was minimal, as the absolute uncertainty (± 0.25 mm) gave a total uncertainty on f which was never more than $\pm 0.5\%$: this is normally true for all microchannels for which the length is larger than 10 mm, and can be made negligible also for shorter microchannels provided the length are measured with more precise instrumentation.

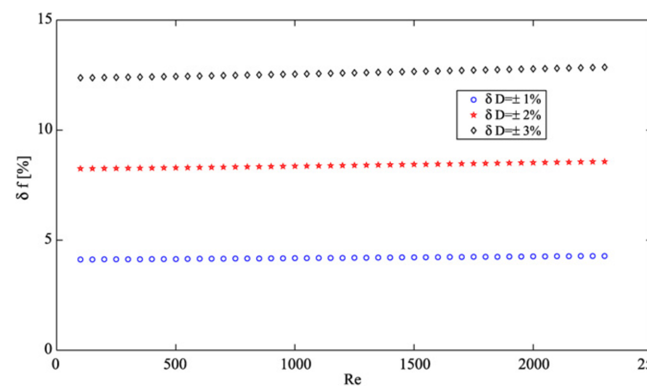


Fig. 4. Influence of the uncertainty of D on f.

From the above considerations, it is possible to conclude that the uncertainties on length, outlet pressure and temperature of the fluid play a minor role on the overall uncertainty and it is sufficient to measure them with an average precision.

The influence of the uncertainty of the diameter is unfortunately somewhat larger: as can be concluded from Fig. 4, even a value as low as $\pm 1\%$ corresponds to an uncertainty on f which increases slightly from around $\pm 4.1\%$ to $\pm 4.3\%$ over the whole range of Reynolds numbers of the laminar regime. Increasing the uncertainty on the diameter increases that on f in a way which can be considered for all practical purposes as proportional.

The incidence of the pressure measurement is easily understood looking at Fig. 5: for an uncertainty on $\Delta p_m = \pm 0.5\%$ of the full scale the uncertainty on f decreases for increasing Reynolds numbers going from about $\pm 3.7\%$ at $Re = 50$ to a value which is roughly $\pm 1\%$ at high Reynolds numbers. This is coherent with the fact that the uncertainty of the pressure transducer is a percentage of its full-scale value: as the Reynolds number increases, the velocity and flow rate increase too. This causes also an increase in the value of the pressure drop which is measured and the relative error decreases correspondingly, although the minimum value for f remains about twice that of the

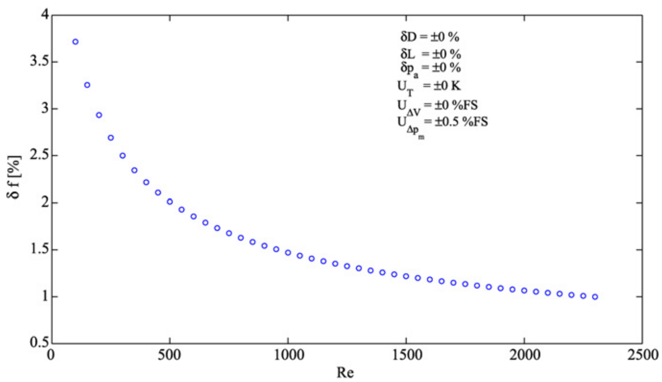


Fig. 5. Influence of the uncertainty of Δp_m on f.

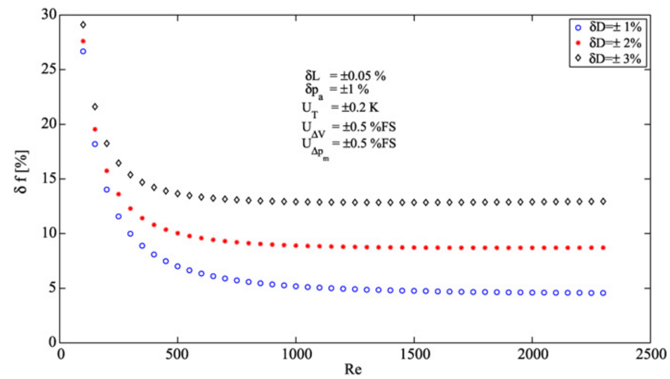


Fig. 7. Influence of the combined uncertainties on f.

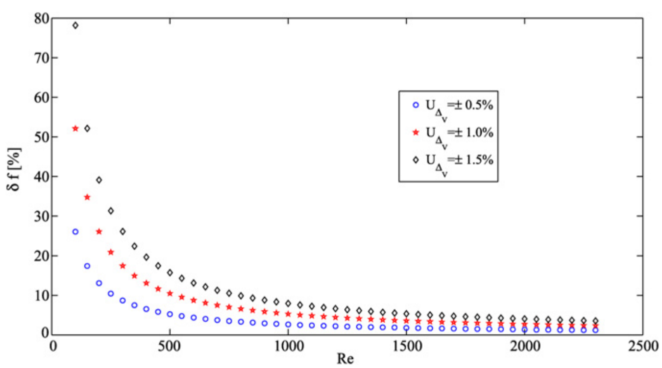


Fig. 6. Uncertainty on f for three different values of $U\dot{V}$.

maximum accuracy of the instrumentation. This effect is due to the way errors propagate.

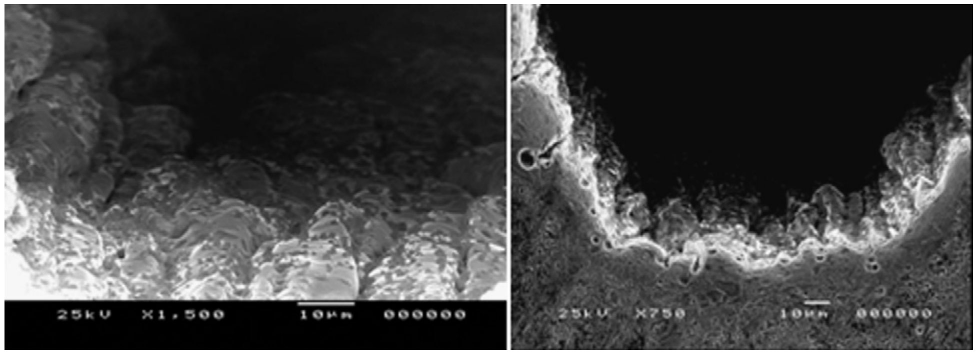
Finally the influence of the uncertainty on the volume flow rate, $\dot{V}$, has been investigated, and the result is shown in Fig. 6. It is to be remarked that even in the case of a reasonably well-chosen range of the meter the low flow rates associated to the lower range of the Reynolds numbers of the study bring very steep increases in the total uncertainty: at $Re = 1000$ and for an instrument with $U\dot{V} = \pm 1.0\%$ FS the uncertainty on f is around $\pm 5.2\%$ and almost doubles at $Re = 500$; a flow meter with $U\dot{V} = \pm 0.5\%$ is needed to halve these values. The geometric characteristics of the microtube are also to be kept in mind: microtubes of smaller diameter may cause a very large variation on the mass flow rate over the whole range of Reynolds numbers in the laminar regime, and this would make thing even worse, should the flow meter have to possess a suitable scale to cover all measurements.

So far, only the contribution of the single uncertainties has been considered, leading to the conclusion that outlet pressure, gas temperature and tube length have a minor influence on the total error on the friction factor and can be confidently measured with standard tools. Concerning the influence of the diameter, its contribution is increasing with the Reynolds number, but so slightly that it can be considered as a constant. Yet, a great accuracy in its determination is mandatory as an uncertainty as small as $\pm 1\%$ becomes more than four-fold in the friction factor.

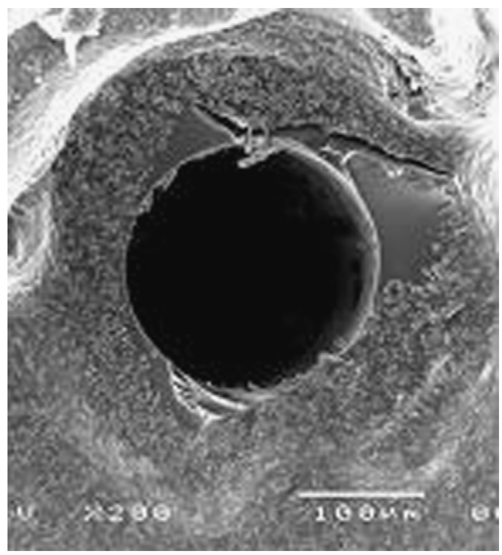
The pressure has a modest and decreasing influence, provided that the instrument's range is chosen suitably. The same trend holds true for flow measurements, but in this case even a careful choice of the measurement range makes it impossible to employ a single meter for the whole span of the Reynolds number in the laminar regime.

The next step is to analyse how all uncertainties combine together in real-life measurements to yield the total uncertainty on f. Again, the same set of error-free, ideal data is produced for the same kind of microtube, and the instrumentation's full scale is as previously employed. The value of the uncertainties chosen for the measured variables are those reported in Table 1, while the results are shown in Fig. 7. As could be expected, at low Reynolds numbers the contribution of the volume flow rate to the total uncertainty is preponderant and almost cancels the other error sources; as the Reynolds number increases, the mass flow rate and pressure drop lose in importance as compared to the effects of the hydraulic diameter, which then becomes the dominant error source. Observing Fig. 7, it can be noticed that for $\delta D = \pm 2\%$ or higher the uncertainty curve exhibits a hard-to spot minimum around Re 900–1000, and then rises again although very slowly. This minimum is the point where the increase in diameter uncertainty starts to overcompensate the decrease in the uncertainty of the other quantities. Another important remark concerning the composite uncertainty is that the minimum value for δf is determined by δD; this means that, especially at moderate to high Reynolds numbers, the accuracy on diameter measurement plays a decisive role in the determination of the friction factor for microtubes, and is not always easy to achieve, especially in the case of stainless steel commercial microtubes as can be highlighted by observing Fig. 8(a) while the fused silica channels are, in general, more regular (Fig. 8(b)). Care should also be exerted when employing commercial microtubes in using the values of the diameter given by the producers; in Fig. 9 the friction factor data are plotted (without error bars to increase readability) in the case of the nominal diameter, D_n, that is 20 μm, and with the real diameter, D_r, 29.9 μm, calculated from the SEM image shown. The difference is obvious, and in this case the error on the diameter would be over 45%, and on a fused silica tube.

When devising experiments, the analysis carried out above can be used to decide beforehand whether the achievable accuracy is bound to yield results which are meaningful; once this has been determined, the minimum accuracy of each instrument or measuring operation can be computed and thus decided



(a)



(b)

Fig. 8. SEM images of commercial stainless steel (a) and peeksil (b) microtubes.

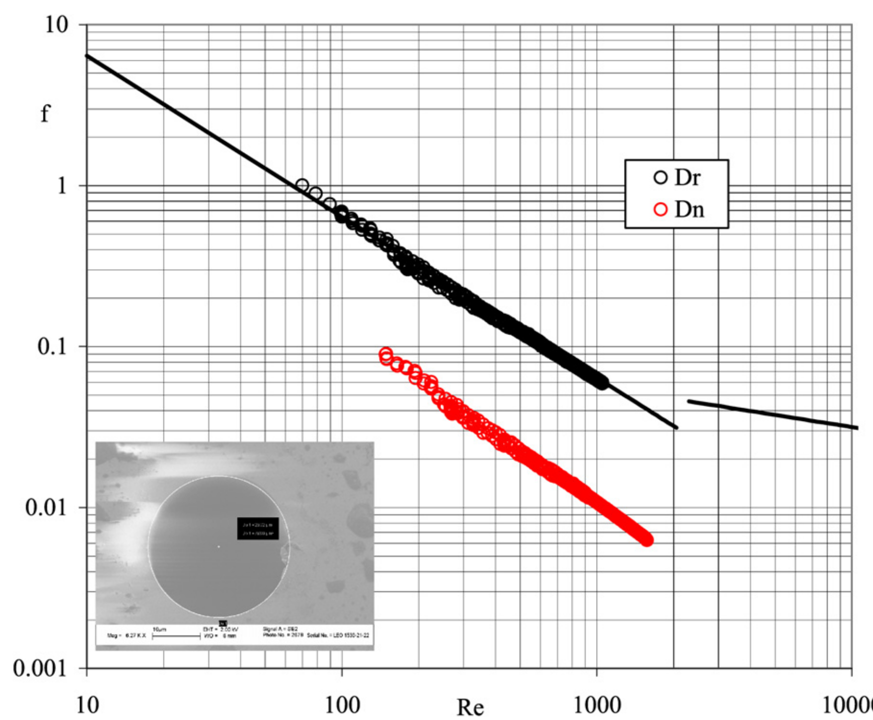


Fig. 9. Friction factor for nominal (D_n) and real (D_r) values of the tube diameter and SEM photograph.

which measurements should be more careful and which instrumentation is worth investing upon. By the same token, if several scales are available for the same instrument (as is the case for pressure transducers or multiple flow meters) the range more suited for the operation can be determined. It is to be remarked that the founding assumption, i.e. that the Poiseuille law holds in the laminar regime, is not a mandatory condition; knowledge of the model to be verified suffices, although the procedure to determine the measured quantities corresponding to an ideal, error-free experiment can be more or less involved.

4. Conclusions

In this work a procedure has been illustrated to assess the limit of validity of experimental data on the friction factor in the laminar regime and to devise experiments for its determination, deciding which instrumentation needs which precision.

To do this, the characteristic length and dimension of the microtube to be tested and the fluid to be employed are chosen first, as are the quantities which should be measured and with which instruments. Once these are known, the model to be verified (in this case the Poiseuille law, but the method is not limited to it) is used to calculate the parameters of interest and the results are used to generate a set of data corresponding to the quantities which ideal instruments would measure in an error-free experiment. These data are then processed so as to determine the influence of each uncertainty when it is the only one present, and then all uncertainties are combined, to simulate the real situation when all errors add up simultaneously. From the case illustrated, it turns out that care has to be exerted in the choice of the scale of the instruments, as this can already abate the maximum uncertainties. If this choice is made properly, the influence of temperature, outlet pressure and tube length measurements are of minor importance, as can be the total pressure drop, if proper actions are taken. The volume flow rate has the greatest influence at low Reynolds numbers, and decreases as the Reynolds number increases. At moderate to high Reynolds numbers, the influence of the uncertainty on the diameter starts to become felt and is the quantity that determines the maximum achievable accuracy on friction factor measurements.

Acknowledgements

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Appendix A

The uncertainty of the friction factor is expressed by Eq. (8), where:

$$A = \left[-2\frac{\alpha}{f}\frac{1-\gamma^2}{(\beta/p_{\text{in}})^2}\right]^2 \tag{A.1}$$

$$B = \left[2\frac{\alpha}{f}\left(1-\frac{\gamma^2}{(\beta/p_{\text{in}})^2}\right)\right]^2 \tag{A.2}$$

$$C = A \tag{A.3}$$

The relative uncertainties on α, β, γ and p_{in} are then to be expressed as a function of the measured quantities, which for α is straightforward:

$$\delta\alpha = \sqrt{(\delta D)^2 + (\delta L)^2}, \tag{A.4}$$

For β the expression is also easily reduced to a function of the basic quantities:

$$\delta\beta = \sqrt{(\delta\dot{m})^2 + (\delta A_c)^2 + \frac{1}{4}(\delta T)^2} \tag{A.5}$$

with

$$\delta\dot{m} = \sqrt{(\delta\rho_n)^2 + (\delta\dot{V})^2} \tag{A.6}$$

$$\delta A_c = \pm 2\delta D \tag{A.7}$$

$$\delta T = \pm\frac{0.2}{T_m(^\circ\text{C})} \tag{A.8}$$

For γ the uncertainties on the inlet and outlet pressure are to be determined as seen below:

$$\delta\gamma = \pm\sqrt{(\delta p_{\text{ex}})^2 + (\delta p_{\text{in}})^2} \tag{A.9}$$

For the outlet pressure

$$\delta p_{\text{ex}} = \pm\sqrt{E(\delta p_2)^2 + F(\delta\beta)^2} \tag{A.10}$$

where

$$E = \left[\frac{p_2}{p_{\text{ex}}}\frac{1}{2}\left(1+\frac{p_2}{\sqrt{p_2^2+2C_{\text{ex}}\beta^2}}\right)\right]^2 \tag{A.11}$$

$$F = \left(\frac{1}{p_{\text{ex}}}\left[\left(\frac{C_{\text{ex}}\beta^2}{\sqrt{p_2^2+2C_{\text{ex}}\beta^2}}\right)\right]\right)^2 \tag{A.12}$$

As to the inlet pressure, considering the case of a compressible flow (which is what has been treated so far):

$$\delta p_{\text{in}} = \pm\sqrt{M(\delta p_1)^2 + N(\delta\beta)^2} \tag{A.13}$$

with

$$M = \left\{\frac{1}{p_{\text{in}}}\left[\frac{p_1}{2}\left(1+\frac{p_1}{\sqrt{p_1^2-2C_{\text{in}}\beta^2}}\right)\right]\right\}^2 \tag{A.14}$$

$$N = \left\{\frac{1}{p_{\text{in}}}\left[\beta\left(\frac{-C_{\text{in}}\beta}{\sqrt{p_1^2-2C_{\text{in}}\beta^2}}\right)\right]\right\}^2 \tag{A.15}$$

$$\delta p_1 = \pm\sqrt{\left(\frac{\Delta p_m}{p_1}\right)^2(\delta\Delta p_m)^2 + \left(\frac{p_{\text{atm}}}{p_1}\right)^2(\delta p_{\text{atm}})^2} \tag{A.16}$$

Finally, the expression of the relative uncertainty, Eq. (8), can be obtained:

$$\delta f = \pm\sqrt{(\delta\alpha)^2 + A(\delta\beta)^2 + B(\delta\gamma)^2 + C(\delta p_{\text{in}})^2} \tag{A.17}$$

As to the controlled variable, the uncertainty associated to it is:

$$\delta Re = \pm\sqrt{\delta\mu^2 + \delta A_c^2 + \delta D^2 + \delta\dot{m}^2} \tag{A.18}$$

The uncertainty on the Reynolds number is a function of all known quantities except the dynamic viscosity, which for the chosen fluid can be expressed by the constitutive equation below:

$$\mu(T) = \mu_a \left[\frac{(T_m + 273.15)}{T_r} \right]^{N_2} \tag{A.19}$$

with an associated uncertainty

$$\delta\mu = \frac{T_m}{\mu} \frac{\mu_0 N_2}{T_n^{N_2}} (T_m + 273.15)^{N_2 - 1} \delta \mathrm{T} \tag{A.20}$$

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